The Hanukkah Four-Dreidel Game

The Hanukkah Four-Dreidel Game: Spin, Score, or Risk It All!

The Hanukkah Four-Dreidel Game is a mathematical adaptation of the dice game Farkle, played with traditional Hanukkah dreidels instead of dice. Each dreidel has four Hebrew letters: Nun, Gimel, Hey, and Shin. Players earn points by spinning dreidels to obtain specific combinations of letters and must decide whether to keep their points or risk losing them by spinning again. The game combines luck, probability, and strategic decision-making.

Rules of the Game

1. The game is designed for 2–8 players.
2. **Winning the Game:** When a player reaches 10,000 points or more, the current round continues until every player has had an equal number of turns. For example, if the second player in a game of four players reaches 10,000 points, the third and fourth players still take their turns, but the first player does not get another turn. The player with the highest total score at the end of the round wins.
3. Each player receives four dreidels. The four Hebrew letters on the dreidels are Nun (נ), Gimel (ג), Hey (ה), and Shin (ש).
4. On each turn, a player spins all four dreidels, hoping to obtain at least one of the scoring combinations shown in the table below.
5. If a player obtains one or more scoring combinations, they must choose at least one combination to keep and set those dreidels aside. They may keep additional scoring combinations if they wish. **Each dreidel may be used in only one scoring combination.** A player cannot count the same dreidel twice.
6. After each spin, the player must decide whether to:
    - **Stop** and add the points accumulated during that turn to their total score.

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    - **Continue** by spinning the dreidels that have not been set aside, hoping to earn more points.

7. If a player spins the remaining dreidels and does not obtain any scoring combination, they get a **Farkle**. They lose all the points accumulated during that turn, and their turn ends. Their total score from previous turns is not affected.
8. A player may not combine dreidels from different spins to form a scoring combination. For example, suppose a player keeps two Gimels from one spin and obtains another Gimel on the next spin. The player cannot combine these three Gimels to claim 1,000 points. The new Gimel is worth only 100 points.
9. **Players may choose between different scoring combinations.** For example, if a player spins four Gimels, they may:
    - Keep all four Gimels for 1,500 points.
    - Keep just three Gimels for 1,000 points and spin the fourth dreidel again.

Similarly, a player who spins three Gimels and one Nun may keep all four dreidels for 1,050 points or keep only the three Gimels for 1,000 points and spin the remaining dreidel again.

10. **Hot Dreidels:** If a player sets aside all four dreidels as scoring combinations, they may either stop and bank all the points accumulated during that turn or continue playing. If they choose to continue, all four dreidels become *hot dreidels*, and the player must spin all four again. If the player gets a Farkle, they lose all the points accumulated during that turn, including those earned before the hot dreidels were spun.
11. A player's turn ends when they either bank their points or get a Farkle. The next player then begins their turn by spinning all four dreidels.

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Scoring Table

Dreidel Result	Points
One Nun (נ)	50
One Gimel (ג)	100
Three Nuns (נ נ נ)	500
Three Gimels (ג ג ג)	1,000
Three Heys (ה ה ה)	450
Three Shins (ש ש ש)	450
One of each letter (נ ג ה ש)	400
Four identical letters	1,500

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Examples of Scoring

Example 1: Four Gimels (ג ג ג ג)

A player may keep all four dreidels for 1,500 points or keep just three Gimels for 1,000 points and spin the remaining dreidel again.

Example 2: One of Each Letter (נ ג ה ש)

A player may keep all four dreidels for 400 points or keep Nun and Gimel for 150 points and spin the other two dreidels again.

Example 3: Three Heys and One Gimel (ה ה ה ג)

A player may keep all four dreidels for 550 points or keep only the three Heys for 450 points and spin the remaining dreidel again.

Example 4: Combinations from Different Spins

A player spins four dreidels and gets two Gimels, one Hey, and one Shin (ג ג ה ש). The player keeps the two Gimels for 200 points.

On the next spin, the remaining two dreidels show Gimel and Nun (ג נ), earning another 150 points.

The player now has 350 points for the turn. They cannot combine the new Gimel with the two Gimels from the previous spin to claim three Gimels.

Remember: The more times you spin, the more opportunities you have to earn points—but one Farkle will cost you all the points accumulated during that turn!

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Hanukkah Farkle

Mathematical Challenges

In Hanukkah Farkle, players spin dreidels and try to obtain combinations listed in the scoring table.

A **Farkle** occurs when none of the scoring combinations in the table appears. The player receives no points and loses all the points accumulated during that turn.

Assume that each dreidel is equally likely to land on any of the four Hebrew letters: Nun (נ), Gimel (ג), Hey (ה), or Shin (ש). Each letter has a probability of 1/4.

Questions

1. What is the probability of getting a Farkle when spinning just one dreidel?

2. What is the probability of getting a Farkle when spinning two dreidels?

3. What is the probability of getting a Farkle when spinning three dreidels?

4. What is the probability of getting a Farkle when spinning four dreidels?

5. Exactly Two Nuns

Four dreidels are spun. What is the probability that exactly two show Nun (נ)?

6. Four Different Letters

Four dreidels are spun. What is the probability of obtaining one of each letter, earning 400 points?

7. Three Identical Letters

Four dreidels are spun. What is the probability that exactly three dreidels show the same letter and the fourth shows a different letter?

8. Comparing Scoring Combinations

When four dreidels are spun, which is more likely: obtaining four identical letters or obtaining one of each of the four letters? How many times more likely?

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9. Conditional Probability

Four dreidels are spun, and none shows Nun or Gimel. Given this information, what is the probability that the player gets a Farkle?

10. Should You Take The Risk?

A player has accumulated 200 points during their turn and has just one dreidel left to spin. The player can either bank the 200 points or spin the remaining dreidel.

Assuming the player will stop after this spin if they score, which choice gives the higher expected number of points? What if the player has accumulated only 50 points? What is the minimum number of points a player should have accumulated before it becomes better to stop rather than spin the remaining dreidel?

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Answers and Explanations

1. One Dreidel

A Farkle occurs if the dreidel lands on Hey or Shin, since neither letter scores points by itself.

There are 2 unfavorable outcomes out of 4 equally likely outcomes.

Answer: 2/4 = 1/2 = 50%

2. Two Dreidels

Both dreidels must show either Hey or Shin. Each dreidel has a probability of 1/2 of showing one of these letters.

Answer: $(1/2)^2 = 1/4 = 25\%$

3. Three Dreidels

All three dreidels must show either Hey or Shin, but three Heys and three Shins are scoring combinations.

There are $2^3 = 8$ sequences containing only Hey and Shin. Two of these score points (HHH and SSS).

Therefore, 6 of the 64 equally likely outcomes produce a Farkle.

Answer: 6/64 = 3/32 = 9.375%

4. Four Dreidels

For a Farkle, all four dreidels must show either Hey or Shin, without forming a scoring combination.

There are $2^4 = 16$ sequences containing only Hey and Shin.

Of these, 2 contain four identical letters, and 8 contain exactly three of one letter and one of the other. All 10 score points.

The remaining 6 sequences contain two Heys and two Shins and produce a Farkle.

Answer: 6/256 = 3/128 ≈ 2.34%

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5. Exactly Two Nuns

First, choose which two of the four dreidels show Nun. There are 6 ways to do this.

Each of the other two dreidels can show any of the remaining three letters.

The number of favorable outcomes is:

$6 \times 3^2 = 54$

There are $4^4 = 256$ equally likely outcomes.

Answer: 54/256 = 27/128 ≈ 21.09%

6. Four Different Letters

The four different letters can appear in any order.

There are 4! = 24 arrangements.

There are 256 equally likely outcomes.

Answer: 24/256 = 3/32 = 9.375%

7. Three Identical Letters

There are 4 choices for the letter that appears three times.

There are 3 choices for the different letter.

There are 4 possible positions for that different letter.

The number of favorable outcomes is:

$4 \times 3 \times 4 = 48$

Answer: 48/256 = 3/16 = 18.75%

8. Comparing Scoring Combinations

Four identical letters can occur in 4 ways:

NNNN, GGGG, HHHH, SSSS.

Four different letters can occur in 4! = 24 ways.

Therefore, obtaining four different letters is more likely.

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24 ÷ 4 = 6

Answer: One of each letter is 6 times as likely as four identical letters.

9. Conditional Probability

We know that none of the four dreidels shows Nun or Gimel. Therefore, each dreidel must show either Hey or Shin.

There are $2^4 = 16$ equally likely outcomes under this condition.

A Farkle occurs only when exactly two dreidels show Hey and exactly two show Shin.

There are 6 ways to choose the positions of the two Heys.

Answer: 6/16 = 3/8 = 37.5%

10. Should You Take The Risk?

With 200 points already accumulated, spinning gives an expected value of

(250 + 300 + 0 + 0)/4 = 137.5, so **banking 200 is better**. With 50 points, spinning gives (100 + 150 + 0 + 0)/4 = 62.5, **so spinning is better**. This assumes the player stops after a scoring result. 75 points is the point of indifference.

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